

# A Holographic Prediction of the Deconfinement Temperature

Christopher P. Herzog

*Physics Department, University of Washington  
Seattle, WA 98195-1560*

(Dated: August 25, 2006)

We argue that deconfinement in AdS/QCD models occurs via a first order Hawking-Page type phase transition between a low temperature thermal AdS space and a high temperature black hole. Such a result is consistent with the expected temperature independence, to leading order in  $1/N_c$ , of the meson spectrum and spatial Wilson loops below the deconfinement temperature. As a byproduct, we obtain model dependent deconfinement temperatures  $T_c$  in the hard and soft wall models of AdS/QCD. Our result for  $T_c$  in the soft wall model is close to a recent lattice prediction.

PACS numbers: 11.25.Tq, 25.75.Nq

## I. INTRODUCTION

Since the discovery of the Anti de Sitter space – Conformal Field Theory (AdS/CFT) correspondence [1, 2, 3] relating type IIB string theory on  $AdS_5 \times S^5$  and  $\mathcal{N} = 4$  super Yang-Mills (SYM) theory, many have hoped that generalizations might yield deep insights into QCD and the nature of confinement. Indeed, many qualitative advances in understanding have occurred. The paper [4] studied confinement in  $\mathcal{N} = 4$  SYM by placing the gauge theory on a sphere and related deconfinement to a Hawking-Page phase transition in the dual gravitational description. The papers [5, 6, 7] realized confinement in related supersymmetric field theories by finding gravitational descriptions that capped off the geometry in a smooth way in the infrared at small radius. The gravitational descriptions in these last three papers are cumbersome, but the geometric insight is clear: cutting off the geometry at small radius produces confinement in the dual gauge theory. Based on this insight, Ref. [8] studied a far simpler model:  $AdS_5$  where the small radius region is removed. While such a removal is brutal, subsequent work has shown that one gets realistic, semi-quantitative descriptions of low energy QCD [9, 10].

In this letter, we perform a semi-quantitative analysis of the deconfinement phase transition in these AdS/QCD models. In particular, we consider both the hard wall model of Refs. [9, 10] and also the soft wall model of Ref. [11] where the authors study a more gentle infrared truncation of  $AdS_5$  induced by a dilaton-like field. This soft wall model has the advantage of producing mesons with a stringy mass spectrum (as compared with the free particle in a box spectrum of the hard wall model).

We find the deconfinement phase transition corresponds to a Hawking-Page [12] type first order transition between thermal AdS at low temperature and an asymptotically AdS geometry containing a black hole at high temperature. A careful look at the gravitational free energies of the cut-off thermal AdS and the black hole solution reveal that the cut-off thermal AdS is stable for a range of temperatures where the black hole horizon radius would appear inside the AdS cavity.

Perhaps based on the observation that  $T_c \rightarrow 0$  for  $\mathcal{N} =$

4 SYM on a sphere as the radius of the sphere goes to infinity, many authors have assumed (see for example [13, 14, 15]) that the black hole phase in these hard and soft wall models is always stable. This assumption leads to physics inconsistent with our expectations of large  $N_c$  gauge theories, as we discuss at the end of the letter. While our argument for a phase transition makes certain subtle assumptions about the gravitational action, the existence of this phase transition is fully compatible with our field theory understanding.

At a time where increasingly researchers are trying to apply AdS/CFT inspired models and calculations to experiment, investigating the consistency and universal features of these models is of critical importance. To cite two better known examples, Ref. [16] has attempted to use the low value of the viscosity to entropy density ratio in these and related models to explain high values of the elliptic flow in heavy ion collisions at RHIC. The Refs. [17] attempt to measure the energy loss rate of heavy quarks from AdS/CFT to gain a better understanding of charm and bottom physics at RHIC. In the absence of a gravity dual for QCD, one approach to experiment is to seek out universal behavior in the gravity duals we do understand; in both models we study, we see evidence for a first order phase transition. While AdS/CFT remains a conjecture, it is important to check that these dual models are consistent with field theory understanding; the phase transition we find is fully compatible with our large  $N_c$  field theory expectations, as we argue at the end.

As a byproduct of our analysis, we relate the mass of vector mesons to the deconfinement temperature. The vector mesons correspond to cavity modes in the cut-off AdS. By matching the mass of the lightest vector meson to experimental data, we can fix the infrared cut-off scale. The Hawking-Page analysis then relates this cut-off scale to the deconfinement temperature. Our prediction of  $T_c \approx 191$  MeV for the soft wall model is amusingly close to one recent lattice prediction [26].

We begin in Section II by performing an analysis of the Hawking-Page phase transition for the hard and soft wall AdS/QCD models. In Section III, we review some results of [9, 10, 11] for vector meson masses in these models to extract a prediction for the deconfinement temperature.

Section IV concludes with remarks about temperature independence of equilibrium quantities in the confining phase at a large number of colors  $N_c$ .

## II. HAWKING-PAGE ANALYSIS

### A. The Hard Wall Model

We establish a relationship between the confinement temperature and the infrared cut-off for the hard wall model assuming that the thermodynamics is governed by the gravitational part of the action. The assumption is justified at large  $N_c$  where the gravitational part scales as  $N_c^2$  while the contribution from the mesons we consider later scales only as  $N_c$ .

We consider a gravitational action of the form

$$I = -\frac{1}{2\kappa^2} \int d^5x \sqrt{g} \left( R + \frac{12}{L^2} \right). \quad (1)$$

The gravitational coupling scales as  $\kappa \sim g_s \sim 1/N_c$ . There are two relevant solutions to the equations of motion. The first is cut-off thermal AdS with a line element

$$ds^2 = L^2 \left( \frac{dt^2 + d\vec{x}^2 + dz^2}{z^2} \right) \quad (2)$$

where the radial coordinate extends from the boundary of AdS  $z = 0$  to a cut-off  $z = z_0$ . The second solution is cut-off AdS with a black hole with the line element:

$$ds^2 = \frac{L^2}{z^2} \left( f(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right), \quad (3)$$

where  $f(z) = 1 - (z/z_h)^4$ . The Hawking temperature of the black hole solution is  $T = 1/(\pi z_h)$ .

In both cases, we continued to Euclidean signature with a compact time direction. In the black hole case, the periodicity is enforced by regularity of the metric near the horizon,  $0 \leq t < \pi z_h$ . In thermal AdS, the periodicity of  $t$  is not constrained.

In either case, the curvature of the solution is  $R = -20/L^2$  and so on-shell, the gravitational action becomes

$$I = \frac{4}{L^2 \kappa^2} \int d^5x \sqrt{g}, \quad (4)$$

i.e. the volume of space-time times a constant [27].

The value of  $I$  for both space-times is infinite, so we regularize by integrating up to an ultraviolet cut-off  $z = \epsilon$ . (We divide out by the trivial infinity related to the integral over the spatial components  $\vec{x}$  of the metric.) For thermal AdS, the regularized action density becomes

$$V_1(\epsilon) = \frac{4L^3}{\kappa^2} \int_0^{\beta'} dt \int_\epsilon^{z_0} dz z^{-5}, \quad (5)$$

while for the black hole in AdS, the density is

$$V_2(\epsilon) = \frac{4L^3}{\kappa^2} \int_0^{\pi z_h} dt \int_\epsilon^{\min(z_0, z_h)} dz z^{-5}. \quad (6)$$

These  $V_i$  are free energy densities in the field theory.

We compare the two geometries at a radius  $z = \epsilon$  where the periodicity in the time direction is locally the same. In other words,  $\beta' = \pi z_h \sqrt{f(\epsilon)}$ . After this adjustment,

$$\begin{aligned} \Delta V &= \lim_{\epsilon \rightarrow 0} (V_2(\epsilon) - V_1(\epsilon)) \\ &= \begin{cases} \frac{L^3 \pi z_h}{\kappa^2} \frac{1}{2z_h^4} & z_0 < z_h \\ \frac{L^3 \pi z_h}{\kappa^2} \left( \frac{1}{z_0^4} - \frac{1}{2z_h^4} \right) & z_0 > z_h \end{cases}. \end{aligned} \quad (7)$$

When  $\Delta V$  is positive (negative), thermal AdS (the black hole) is stable. Thus the Hawking-Page phase transition occurs at a temperature corresponding to  $z_0^4 = 2z_h^4$ , or

$$T_c = 2^{1/4}/(\pi z_0). \quad (8)$$

As the temperature increases, thermal AdS becomes unstable and the black hole becomes stable. At  $T_c$ , the black hole horizon forms inside the AdS cavity, between the boundary and the infrared cut-off, at a radius  $z_h < z_0$ .

### B. The Soft Wall Model

The calculation for the soft wall model of [11] is similar to the hard wall model calculation described previously. Certain features require additional explanations and assumptions. In place of (1), we have the action

$$I = -\frac{1}{2\kappa^2} \int d^5x \sqrt{g} e^{-\Phi} \left( R + \frac{12}{L^2} \right) \quad (9)$$

where  $\Phi = cz^2$  is a dilaton like field taken to have non-trivial expectation value. This dilaton field is assumed not to affect the gravitational dynamics of our theory. As in [11], we assume that AdS space solves the equations of motion for the full theory. We make the additional assumption that the black hole in AdS (3) also satisfies the equations of motion. The on-shell action is then (4) scaled by a dilaton dependent factor:

$$I = \frac{4}{L^2 \kappa^2} \int d^5x \sqrt{g} e^{-\Phi}. \quad (10)$$

To trust this set of assumptions, we should construct an explicit supergravity background with these properties, something we have not done. Such a solution may exist. In string frame, the dilaton kinetic factor has the wrong sign, and it is conceivable one may construct a nontrivial solution with a trivial stress energy tensor. For example the dilaton-tachyon system considered by [22] has precisely such a solution with a quadratic dilaton and linear tachyon but also breaks Lorentz invariance [23]. Regardless of its quantitative significance, qualitatively our answer must be right because it conforms with our large  $N_c$  field theory expectations, as we argue at the end.

From this on-shell action (10), we calculate the regularized action densities for thermal AdS:

$$V_1(\epsilon) = \frac{4L^3}{\kappa^2} \int_0^{\beta'} dt \int_\epsilon^\infty dz z^{-5} e^{-cz^2}, \quad (11)$$

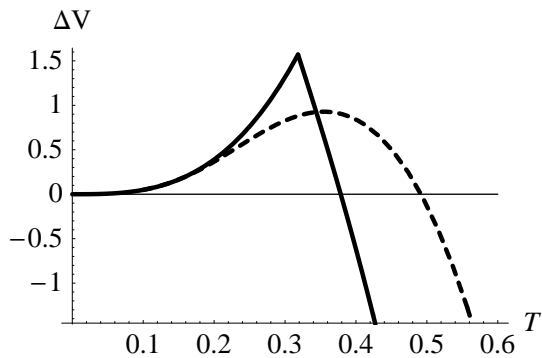


FIG. 1: The solid line is the free energy difference in the hard wall model, the dashed line the difference in the soft wall model. The graph was made in units where  $L^3/\kappa^2 = c = z_0 = 1$ .

and for the black hole solution

$$V_2(\epsilon) = \frac{4L^3}{\kappa^2} \int_0^{\pi z_h} dt \int_{\epsilon}^{z_h} dz z^{-5} e^{-cz^2}. \quad (12)$$

Choosing  $\beta'$  as in the hard wall model,  $\beta' = \pi z_h \sqrt{f(\epsilon)}$ , the end result is

$$\begin{aligned} \Delta V &= \lim_{\epsilon \rightarrow 0} (V_2 - V_1) \\ &= \frac{\pi L^3}{\kappa^2 z_h^3} \left( e^{-cz_h^2} (-1 + cz_h^2) + \frac{1}{2} + c^2 z_h^4 \text{Ei}(-cz_h^2) \right). \end{aligned} \quad (13)$$

Here,  $\text{Ei}(x) \equiv -\int_{-x}^{\infty} e^{-t}/t dt$ . Numerically, there will be a phase transition from thermal AdS to the black hole solution when  $cz_h^2 = 0.419035\dots$ , or

$$T_c = 0.491728\sqrt{c}. \quad (14)$$

For small temperatures (large  $z_h$ ),  $\Delta V \rightarrow L^3\pi/(2\kappa^2 z_h^3) > 0$  and thermal AdS is stable. For large temperatures (small  $z_h$ ),  $\Delta V \rightarrow -L^3\pi/(2\kappa^2 z_h^3) < 0$  and the black hole solution is stable.

### III. VECTOR MESONS AND MATCHING QCD

In the previous section, we found expressions (8) and (14) that related the deconfinement temperature to, in one case, the infrared hard wall cut-off  $z_0$  and, in the other, the soft wall parameter  $c$ . Here we review results from Refs. [9, 10, 11, 20] that relate  $z_0$  and  $c$  to the spectrum of vector mesons in QCD and thus get a relation between the mass of the lightest vector meson and the deconfinement temperature.

In the hard and soft wall cases, Refs. [9, 10, 11] model vector mesons as cavity modes of a vector field in this modified AdS space. Choosing a radial gauge where  $V_z = 0$ , these vector fields  $V_\mu(x, z) = V_\mu(q, z)e^{iq \cdot x}$  satisfy the equation of motion

$$\partial_z \left( \frac{1}{z} e^{-\Phi} \partial_z V_\mu(q, z) \right) - \frac{e^{-\Phi} q^2}{z} V_\mu(q, z) = 0, \quad (15)$$

where  $\Phi = 0$  in the hard wall model and  $\Phi = cz^2$  in the soft wall model.

In the hard wall case, normalizable boundary conditions at  $z = 0$  determine that the solutions are Bessel functions:  $V_\mu(q, z) \sim z J_1(mz)$ , where  $m^2 = -q^2$ . Applying Neumann boundary conditions at the cut-off  $z = z_0$ , one finds only a discrete set of eigenmodes corresponding to discrete choices of  $q$  which satisfy  $J_0(m_i z_0) = 0$ . The first zero of  $J_0(x)$  occurs at  $x = 2.405\dots$ , implying the lightest  $\rho$  meson has a mass  $m_1 = 2.405/z_0$ . Experimentally, the lightest  $\rho$  meson has a mass of  $m_1 = 776$  MeV. Thus, we conclude that  $z_0 = 1/(323 \text{ MeV})$ .

We now can make a prediction for the deconfinement temperature in this hard wall model:

$$T_c = 2^{1/4}/(\pi z_0) \approx 0.1574 m_\rho = 122 \text{ MeV}, \quad (16)$$

a low number compared to new lattice estimates [18, 19].

In the soft wall model, the relevant solution to this differential equation (15) involves Laguerre polynomials:  $V_\mu(q, z) \sim z^2 L_n^1(cz^2)$  where the allowed values of  $q$  are  $-q^2 = 4nc$  ( $n \in \mathbb{Z}^+$ ). Matching to the lightest  $\rho$ -meson, we find  $\sqrt{c} = 338$  MeV. Our prediction for the deconfinement temperature in the soft wall model is thus

$$T_c = 0.4917\sqrt{c} \approx 0.2459 m_\rho = 191 \text{ MeV} \quad (17)$$

which is a current lattice prediction [18]. Because phase transitions are sensitive to the density of states, perhaps the fact that the soft wall model produces a more realistic meson spectrum is related to this improved prediction compared with the hard wall model.

### IV. DISCUSSION

We argue that the stability of thermal AdS at low temperatures and the presence of a first order phase transition in these soft and hard wall models of QCD is consistent with large  $N_c$  field theory expectations. Recall at large  $N_c$ , the confining low temperature phase has  $O(1)$  entropy density, discrete meson and glueball spectra, and vanishing expectation value for the Polyakov loop (a time like Wilson loop). The deconfined, high temperature phase has  $O(N_c^2)$  entropy density, temperature dependent spectral densities, and a non-zero expectation value for the Polyakov loop.

From these properties, it follows that in soft wall models, the black hole configuration cannot be stable for all temperatures. The soft wall model is intended to be a model of QCD which experiences a phase transition at  $T_c > 0$ , but the presence of a horizon at  $T = 0$  indicates that  $T_c = 0$ . In the gravity dual, the presence of a horizon introduces  $O(N_c^2)$  degrees of freedom. Mesons [11] and glueballs in this soft-wall model correspond to discrete cavity modes, but the presence of a horizon leads to loss of probability density into the black hole and smears out the spectrum. Moreover, with a horizon, there is no obvious topological reason for the Polyakov loop to

vanish [4]. In the hard wall model, the sharp cut-off at  $z = z_0$  can completely hide the horizon and the preceding horizon dependent arguments fail. However, even if the cut-off at  $z_0 < z_h$  completely cloaks the horizon, the metric far from the horizon is still different in the black hole background and leads to spatial Wilson loops and a mass spectrum for mesons and glueballs inconsistent with our large  $N_c$  expectations, as we now argue.

In the confined phase, spatial Wilson loops have an expectation value that to leading order in  $1/N_c$  is temperature independent. From the field theory perspective, a spatial Wilson loop produces a sheet of flux that sits at a point in the compactified time direction. Temperature dependence can only come from fluctuations that are large compared to the inverse temperature scale, wrap around the compactified time direction, and lead to self intersections of the flux sheet. These self intersections are suppressed by a power of  $1/N_c$ . From the gravitational perspective, these spatial Wilson loops correspond to Euclidean string world sheets that droop from the boundary of AdS toward the center. The area law comes from the fact that for a large enough boundary, most of the string world sheet lies along the cut-off. If the black hole configuration were always stable, even though the horizon is hidden behind the cut-off, the string would experience temperature dependent curvature corrections that alter its effective tension. Instead, since thermal AdS is thermodynamically preferred, the expectation value will be temperature independent.

Next, consider the mesons and glueballs which in these AdS/QCD models correspond to cavity modes [9, 10, 11, 21]. Again, if the black hole solution were always stable, even if the event horizon were effectively hidden by the cut-off, temperature dependent curvature

corrections would appear in the mass spectrum. Such corrections are not expected from the point of view of large  $N_c$  field theory. In the confining phase, the interaction cross sections of mesons and glue balls are  $1/N_c$  suppressed. From chiral perturbation theory, for example, Ref. [24] demonstrated that temperature dependent corrections to meson masses involve diagrams with at least two pions in the intermediate channel. Since the decay width to pions is already  $1/N_c$  suppressed, the mass corrections must be as well. More formally, we could integrate out the fermions from our theory and re-express the mass corrections for mesons in terms of sums of non-local operators, for example, the spatial Wilson loops discussed above which have  $1/N_c$  suppressed temperature corrections [25] [28].

In conclusion, we emphasize that the formation of a black hole in these AdS/QCD models does not happen at  $T = 0$ , but is instead dual to a first order deconfinement phase transition at a finite temperature  $T_c > 0$ . A black hole at  $T = 0$  would lead to  $\mathcal{O}(N_c^2)$  entropy density, a nonvanishing Polyakov loop, leading order in  $1/N_c$  temperature corrections to spatial Wilson loops, and other effects inconsistent with our expectations for the confining phase of a large  $N_c$  gauge theory.

#### Acknowledgments

I would like to thank Andreas Karch, Berndt Mueller, Steve Sharpe, Aleksi Vuorinen, and Larry Yaffe for useful discussions and for encouraging me to write up these observations. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-96ER40956.

- 
- [1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].
  - [2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109].
  - [3] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].
  - [4] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998) [arXiv:hep-th/9803131].
  - [5] I. R. Klebanov and M. J. Strassler, JHEP **0008**, 052 (2000) [arXiv:hep-th/0007191].
  - [6] J. M. Maldacena and C. Nunez, Phys. Rev. Lett. **86**, 588 (2001) [arXiv:hep-th/0008001].
  - [7] J. Polchinski and M. J. Strassler, arXiv:hep-th/0003136.
  - [8] J. Polchinski and M. J. Strassler, Phys. Rev. Lett. **88**, 031601 (2002) [arXiv:hep-th/0109174].
  - [9] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005) [arXiv:hep-ph/0501128].
  - [10] L. Da Rold and A. Pomarol, Nucl. Phys. B **721**, 79 (2005) [arXiv:hep-ph/0501218].
  - [11] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006) [arXiv:hep-ph/0602229].
  - [12] S. W. Hawking and D. N. Page, Commun. Math. Phys. **87**, 577 (1983).
  - [13] K. Ghoroku and M. Yahiro, Phys. Rev. D **73**, 125010 (2006) [arXiv:hep-ph/0512289].
  - [14] H. Boschi-Filho, N. R. F. Braga and C. N. Ferreira, arXiv:hep-th/0607038.
  - [15] O. Andreev and V. I. Zakharov, arXiv:hep-ph/0607026.
  - [16] E. Shuryak, Prog. Part. Nucl. Phys. **53**, 273 (2004) [arXiv:hep-ph/0312227].
  - [17] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L. G. Yaffe, JHEP **0607**, 013 (2006) [arXiv:hep-th/0605158].
  - [18] S. S. Gubser, arXiv:hep-th/0605182.
  - [19] J. Casalderrey-Solana and D. Teaney, Phys. Rev. D **74**, 085012 (2006) [arXiv:hep-ph/0605199].
  - [20] F. Karsch, arXiv:hep-lat/0608003.
  - [21] Y. Aoki, Z. Fodor, S. D. Katz and K. K. Szabo, arXiv:hep-lat/0609068.
  - [22] O. Andreev, Phys. Rev. D **73**, 107901 (2006) [arXiv:hep-th/0603170].
  - [23] H. Boschi-Filho and N. R. F. Braga, Eur. Phys. J. C **32**, 529 (2004) [arXiv:hep-th/0209080].

- [22] O. Bergman and S. S. Razamat, arXiv:hep-th/0607037.
- [23] A. Karch, personal communication.
- [24] V. L. Eletsky and B. L. Ioffe, Phys. Rev. D **51**, 2371 (1995) [arXiv:hep-ph/9405371].
- [25] P. Kovtun, M. Ünsal, and L. G. Yaffe, to appear.
- [26]  $T_c = 191$  MeV comes from [18]. Note however a more recent result [19] arguing that the crossover transition is broad of order 150 to 170 MeV and depends on the observable chosen as order parameter.
- [27] There is a surface term, but, as noted by [12], the surface term contribution from  $z \rightarrow 0$  vanishes for these space-times. In principle, there could be a surface term from  $z = z_0$  as well. In the spirit of [5, 6] where space-time ends at  $z = z_0$  without a boundary, it seems best to set the Gibbons-Hawking term at  $z = z_0$  to zero by hand.
- [28] I would like to thank Larry Yaffe for discussion of these issues.